3. Dependability Evaluation

- How good are all these redundancy techniques, really ??
- We need proper models for a statistical analysis

3.1 Definitions

- Failure rate
  expected number of failures of a type of system over a given period of time
  - denoted: λ
    when a constant value can be assumed

Definitions

Reliability R(t)

- Conditional probability that component operates through (t_o, t), given it was operating correctly at t_o
- Suppose we have N identical components, we run a test on them by putting all of them in operation at t_o and record the number of failed or working components at t.

3.2 Dependability Evaluation

$N_f(t)$: = number of components that have failed at t

$N_o(t)$: = number of components are operating correctly at t

$N = N_f(t) + N_o(t)$

Assumption: once a component fails, it remains failed indefinitely

Reliability the probability that a component has survived time interval [t_o, t]

$$R(t) = \frac{No(t)}{N} = \frac{No(t)}{No(t) + Nf(t)}$$

Dependability Evaluation

Unreliability the probability that a component has not survived time interval $[t_o, t]$

$$Q(t) = \frac{N_f(t)}{N} = \frac{N_f(t)}{N_o(t) + N_f(t)}$$

At any time t: R(t) = 1.0 – Q(t)

because $$R(t) + Q(t) = \frac{N_o(t) + N_f(t)}{N_o(t) + N_f(t)} = 1$$

Dependability Evaluation

Rewrite reliability as a differentiation of R(t) over time

$$R(t) = 1.0 - \frac{N_f(t)}{N}$$

$$\frac{dR(t)}{dt} = -\frac{1}{N} \cdot \frac{dN_f(t)}{dt}$$

$$\underbrace{\frac{dNf(t)}{dt}} = -N \cdot \frac{dR(t)}{dt}$$

this is the instantaneous rate at which components are failing

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Dependability Evaluation

At time t, there are $N_o(t)$ components operational
Normalize by dividing LHS by $N_o(t)$:

$$z(t) = \frac{1}{N_o(t)} \cdot \frac{dN_f(t)}{dt}$$

Hazard function or ***failure rate*** function
"failures per unit of time"

Dependability Evaluation

Failure rate

$$z(t) = \frac{1}{N_o(t)} \cdot \frac{dN_f(t)}{dt} = \frac{1}{N_o(t)} \cdot (-N \frac{dR(t)}{dt}) = -\frac{\frac{dR(t)}{dt}}{R(t)}$$

$$z(t) = -\frac{\frac{dR(t)}{dt}}{R(t)} = \frac{\frac{dQ(t)}{dt}}{1 - Q(t)} \left.\vphantom{\frac{\frac{dQ}{dt}}{1}}\right\} \longleftarrow \text{failure density function}$$

Dependability Evaluation

- Failure rate depends on time
- Practical experiments for (electronic) components show "***bath tub***" curve

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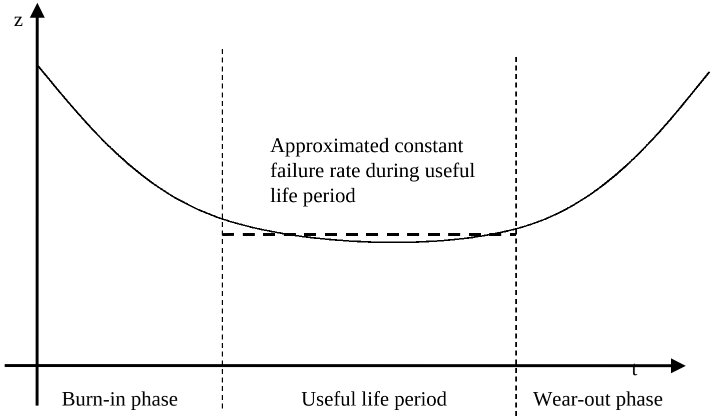
Bathtub Curve



Bathtub Curve

burn-in phase

frequent failures due to substandard or weak components

wear-out phase

after low period of operation increased failure rate due to physical wear of components

useful-life phase

assume $z(t) = \lambda$ (constant percentage for "failures per hour")

Bathtub Curve

- Very common in electronics and mechanical systems: most failures show up very early or very late in the lifetime of a product
- E.g. brand new car ⇔ very old car
  brand new stereo ⇔ very old stereo
- Electronic components spend "burn-in phase" in factory, so customer receives more reliable components

Reliability

Previously:
$$z(t) = \frac{1}{N_o(t)} . \frac{dN_f(t)}{dt} = -\frac{N}{N_o(t)} . \frac{dR(t)}{dt} = -\frac{\frac{dR(t)}{dt}}{R(t)}$$

Differential Equation:
$$\frac{dR(t)}{dt} = -z(t) \cdot R(t)$$

Closed Solution:
$$R(t) = e^{-\int z(t)dt}$$

Reliability

Assume we are within “useful life period”:

$R(t) = e^{-\lambda t}$

- “Exponential failure law” (constant failure rate λ)
- Reliability varies exponentially with time.

Reliability

Caution

- A constant failure rate cannot always be assumed!
- E.g. software faults a time-varying!
  During use, design faults and software faults will be discovered and corrected
  - ➢ failure rate should **decrease** over time
  - ➢ reliability should **increase** over time

Reliability

Modeling time-varying failures: Weibull Distribution

$z(t) = \alpha \cdot \lambda \cdot (\lambda t)^{\alpha - 1}$

$\alpha < 1$: z(t) decreases over time

$\alpha > 1$: z(t) increases over time

$\alpha = 1$: $z(t) = \lambda$, constant

Reliability

$$\longrightarrow \quad \frac{dR(t)}{dt} = -z(t) \cdot R(t) = -\alpha\lambda(\lambda t)^{\alpha-1} R(t)$$

$$\longrightarrow \quad R(t) = e^{-(\lambda t)^{\alpha}}$$

(can be verified by calculating derivative of R(t))

e.g. for $\alpha = -1 \Rightarrow \quad R(t) = e^{-\frac{1}{\lambda t}}$

3.3 Mean Times

- Mean time to failure
- Mean time to repair
- Mean time between failure

Mean Time to Failure

$$MTTF = \sum_{i=1}^{N} \frac{t_i}{N}$$

- N identical systems are placed into operation at t = 0
- We **measure** the time each system operates before failing
- MTTF is average time.

Mean Time to Failure

- MTTF can be **calculated** by finding the expected value of the time of failure.
- Expected value of a random variable x :

$$E[x] = \int_{-\infty}^{+\infty} x \cdot \underbrace{f(x)}_{\text{probability density function}} dx$$

- Apply to expected value of time of failure (= MTTF)

Mean Time to Failure

$$MTTF = \int_0^{+\infty} t \cdot \mathrm{f(t)dt}$$

failure density function

from 0, since failure density is undefined for t<0

From previous definition we know: $f(t) = \frac{dQ(t)}{dt}$

Mean Time to Failure

$$MTTF = \int_0^{\infty} t \cdot \frac{dQ(t)}{dt} dt$$

$$\left(\frac{dQ(t)}{dt} = -\frac{dR(t)}{dt}\right)$$

$$= -\int_0^{\infty} t \cdot \frac{dR(t)}{dt} dt$$

(integration by parts)

$$= \left[-tR(t) + \int R(t)dt\right]_o^{\infty}$$

Mean Time to Failure

$$= \underbrace{-\infty \underbrace{R(\infty)}_{=0}}_{=0} + \underbrace{0 \cdot R(0)}_{=0} + \int_0^{\infty} R(t)dt$$

provided that R (∞) = 0

$$\boxed{MTTF = \int_0^{\infty} R(t)dt}$$

Mean Time to Failure

If “useful life period”: $R(t) = e^{-\lambda t}$

$$\left[-\frac{1}{\lambda} e^{-\lambda t}\right]_0^{\infty} = \frac{1}{\lambda}(-e^{-\lambda\infty} + e^{-\lambda \cdot 0}) = \frac{1}{\lambda}(0+1)$$

$$\boxed{MTTF = \frac{1}{\lambda}}$$

Mean Time to Repair

- MTTR average time to **repair** a system is very difficult to estimate
- If the ith of N faults needs t_i to repair, then:

$$MTTR = \frac{\sum_{i=1}^{N} t_i}{N}$$

Mean Time to Repair

usually assume repair rate μ (number of repairs per hour)

$$MTTR = \frac{1}{\mu}$$

Mean Time Between Failure

- N systems operating for some time T
- Number of failures in system i is n_i
- average number of failures:

$$n_{avg} = \sum_{i=1}^{N} \frac{n_i}{N}$$

$$MTBF = \frac{T}{n_{avg}}$$

- Also: MTBF = MTTF + MTTR

3.4 Reliability Modeling

- System model, concentrating on reliability aspect
- Models:
  - – Combinatorial Models
  - – Markov Models

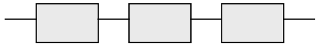
Combinatorial Models

- Probabilistic techniques
- Express reliability of system as a function of reliability of its components
- Construction models:
  - – series
  - – parallel

Combinatorial Models

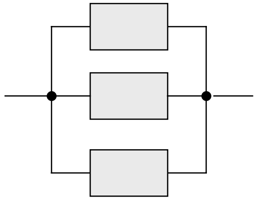
Series

- All components must work correctly
- No redundancy



Parallel

- Only one of the components must work correctly
- High redundancy



Combinatorial Models

N	number of components
$C_{iw}(t)$	represents the event that component C_i is working properly at time t
$R_i(t)$	reliability of component C_i at time t
$R_{series}(t)$	reliability of series system
$R_{parallel}(t)$	reliability of parallel system

Combinatorial Models

SERIES

- Reliability at time t is probability that all N components are working properly

$$R_{series}(t) = P\{C_{1w}(t) \bigcap C_{2w}(t) \bigcap \ldots \bigcap C_{Nw}(t)\}$$

- Assuming the events $C_{iw}(t)$ are ***INDEPENDENT***

$$R_{series}(t) = R_1(t) \cdot R_2(t) \cdot \ldots \cdot R_N(t)$$
$$= \prod_{i=1}^{N} Ri(t)$$

Combinatorial Models

SERIES

- Now for the case of the “exponential failure low”: each C_i has constant failure rate λ_i

$$R_{series}(t) = e^{-\lambda_1 t} \cdot e^{-\lambda_2 t} \cdot \ldots \cdot e^{-\lambda_N t}$$

$$= e^{-\sum_{i=1}^{N} \lambda_i t}$$

Combinatorial Models

PARALLEL

- Only one of N components must function
- Unreliability $Q_{parallel}(t)$, calculated based on event $C_{if}(t)$ that component C_i has failed at time t
  $Q_{parallel}(t) = P\{C_{1f}(t) \bigcap C_{2f}(t) \bigcap .... \bigcap C_{Nf}(t)\}$
- Assuming $C_{if}(t)$ are ***INDEPENDENT***
  $Q_{parallel}(t) = Q_1(t) \cdot Q_2(t) \cdot \ldots \cdot Q_N(t) \quad = \prod_{i=1}^{N} Q_i(t)$

Combinatorial Models

PARALLEL

- $C_{iw}(t) + C_{if}(t) = 1.0$
- $R_{parallel}(t) = 1.0 - Q_{parallel}(t)$

$$= 1.0 - \prod_{i-1}^{N} Q_i(t)$$

$$= 1.0 - \prod_{i=1}^{N} (1.0 - R_i(t))$$

Combinatorial Models

Independence of events

- Can be assumed for random hardware failure
- Cannot be assumed for failures e.g. due to external disturbance
- Therefore: combinational model is most often used for analyzing random failures

Combinatorial Models

M-of-N system

- M of N components required to work
- Can tolerate N–M models failure
- Generalization of several models
  - parallel system: 1-of-N system
  - serial system: N-of-N system
  - TMR: 2-of-3 system
- $R_{MofN}(t) = \sum_{T=0}^{N-M} \binom{N}{i} \cdot \left[R^{N-i}(t) \cdot (1.0 - R(t))^{i} \right]$

$$\binom{N}{i} = \frac{N!}{(N-i)! \cdot i!}$$

Combinatorial Models

R_{MofN} formula covers all possibilities:

- Probability of 0 components failing
- Probability of 1 component failing
- Probability of 2 components failing
- …
- Probability of N–M components failing

Markov Models

- Many complex problems cannot be modeled easily in combinational fashion
- Repair is very difficult to model combinatorially

→ Use Markov models (aka Markov chains)

Markov Models

STATE

- Represents all that must be known to describe system at a given instant in time
- E.g. for reliability: Each state represents a distinct combination of faulty and fault-free modules

TRANSITION

- Changes of state that happen in system
- Over time as failures (reconf.) occur, system goes from one state to another
- State changes are given **probabilites**
  (e.g. prob. of failure, etc.)

Markov Models

Example Markov model of TMR

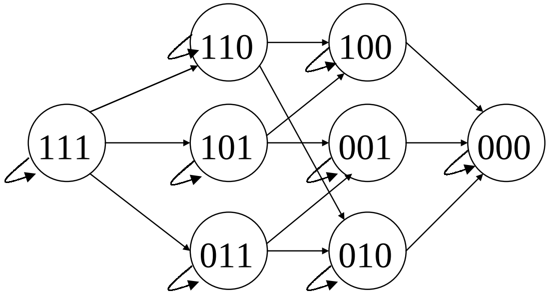
States:

- all possible combinations of operational and failed components
- each state contains bit-string representing component state (e.g. 101, 1 = OK, 0 = fault)

Transitions:

- here only occurrence of fault, no repair

Markov Models



Start state 111

Next state 110, 101 or 011 depending which PE fails

Next state 100, 001 or 010 depending which PE fails

Final state 000

Markov Models

Transition probabilities

(for TMR with exp. failure law: constant failure rate λ)

- Probability of module failed at time t+Δt, given that module was operational at time t:

$$P = 1 - \frac{R(t+\Delta t)}{R(t)} = 1 - \frac{e^{-\lambda(t+\Delta t)}}{e^{-\lambda t}}$$

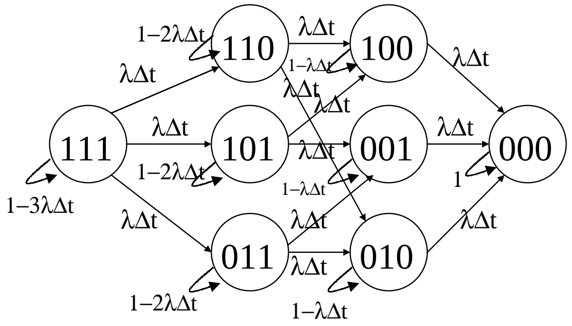
$$= 1 - e^{-\lambda\Delta t} \approx \lambda\Delta t \quad for\ \lambda\Delta t < 1$$

Markov Models

Approximation in more detail:

$$1 - e^{-\lambda\Delta t} = 1 - (1 + (-\lambda\Delta t + \frac{(-\lambda\Delta t)^2}{2!} + \ldots) = \underbrace{+\lambda\Delta t}_{\text{dominant term}} - \frac{(-\lambda\Delta t)^2}{2!} - \ldots$$

Markov Models



Markov Models

- We could now label all transitions with λΔt or the appropriate fraction thereof
- However for practical purposes it doesn’t matter which module fails, but it is more important how many fail.

→ Collapse states into meaningful categories

→ Add individual transition probabilities and divide by number of states to get transition prob. for collapsed states

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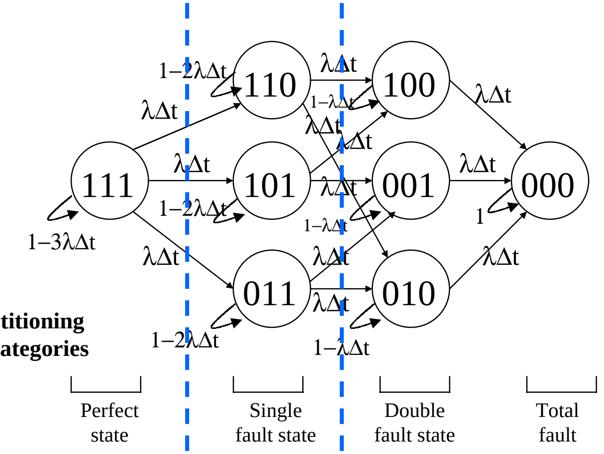
Markov Models



Partitioning in categories

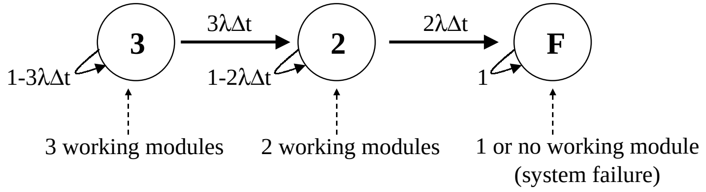
Perfect state

Single fault state

Double fault state

Total fault

Reduced Markov Model



3 working modules

2 working modules

1 or no working module (system failure)

Reduced Markov Model

Extract state transition matrix from chain

from example:

$P_3(t + \Delta\, t) = P_3(t) \cdot P_3(1 - 3\lambda\Delta t)$ *probability of being in state 3*

$P_2(t + \Delta\, t) = P_3(t) \cdot 3\lambda\Delta t + P_2(t) \cdot (1 - 2\lambda\Delta t)$ *p. of being in state 2*

$P_F\,(t + \Delta\, t) = P_2(t) \cdot 2\lambda\Delta t + P_F(t)$ *probability of being in state F*

Reduced Markov Model

- Rewrite as Transition Matrix

$$\begin{bmatrix} P_3(t+\Delta t) \\ P_2(t+\Delta t) \\ P_F(t+\Delta t) \end{bmatrix} = \begin{bmatrix} 1-3\lambda\Delta t & 0 & 0 \\ 3\lambda\Delta t & 1-2\lambda\Delta t & 0 \\ 0 & 2\lambda\Delta t & 1 \end{bmatrix} \bullet \begin{bmatrix} P_3(t) \\ P_2(t) \\ P_F(t) \end{bmatrix}$$

- $P(t + \Delta t) = A \cdot P(t)$

Reduced Markov Model

Equation can be viewed as difference equation:

$P(0) \longleftarrow$ *given (e.g. state 3)*

$P(\Delta t) = A \cdot P(0)$

$P(2\Delta t) = A \cdot P(\Delta t) = A^2 \cdot P(0)$

"

"

$P(n \cdot \Delta t) = A^n \cdot P(0)$

Reduced Markov Model

It is possible to get closed form solutions
for Markov models

$$\frac{dP_3(t)}{dt} = \frac{P_3(t+\Delta t) - P_3(t)}{\Delta t} = \frac{(1-3\lambda\Delta t)P_3(t) - P_3(t)}{\Delta t}$$

$$= -\frac{3\lambda\Delta t P_3(t)}{\Delta t}$$

$$= -3\lambda P_3(t)$$

Reduced Markov Model

$$\frac{dP_2(t)}{dt} = \frac{P_2(t+\Delta t) - P_2(t)}{\Delta t} = 3\lambda P_3\,(t) - 2\lambda P_2(t)$$

$$\frac{dP_F(t)}{dt} = \frac{P(t+\Delta t) - P_F(t)}{\Delta t} = 2\lambda P_2(t)$$

Reduced Markov Model

Closed solutions to differential equations:

$$P_3(t) = e^{-3\lambda t}$$

$$P_2(t) = 3e^{-2\lambda t} - 3e^{-3\lambda t}$$

$$P_F(t) = 1 - 3e^{-2\lambda t} + 2e^{-3\lambda t}$$

Note: Reliability of TMR system is probability of being in states 3 and 2 (this equals 1–P_F)

3.5 Safety Modeling

- Use Markov chains for dependability modeling especially to handle **coverage** in systematic fashion
- Coverage (C) is probability that fault will be handled correctly, given that fault has occurred
- Incorporate coverage in Markov chain
  → model system safety

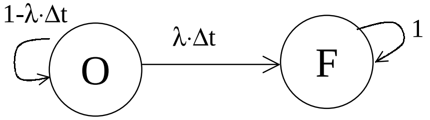
Safety Modeling

- Previously: only systems with perfect coverage, C = 1.0
- Now: $0.0 \le C \le 1.0$
- In Markov chain:
  every un-failed state has 2 transition paths:
  - correct state
  - uncovered state
- Previously: n states (fully covered)
- Now : 2n–1 states (–1 because failed state only once)

Safety Modeling

Example: Simplex-system (only one component)

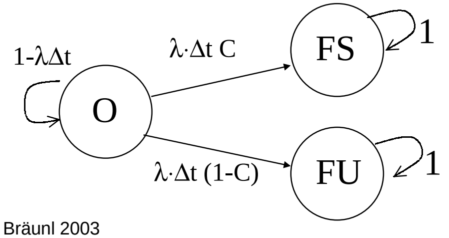
- No safety modeling: 2 states:
  - O operational
  - F failed



Safety Modeling

Example: Simplex-system (only one component)

- With safety modeling: 3 states
  - O fully operational
  - FS failed-safe state
  - FU failed-unsafe state



Safety Modeling

Corresponding transition matrix

$$\begin{bmatrix} P_O(t+\Delta t) \\ P_{FS}(t+\Delta t) \\ P_{FU}(t+\Delta t) \end{bmatrix} = \begin{bmatrix} 1-\lambda\Delta t & 0 & 0 \\ \lambda\Delta t C & 1 & 0 \\ \lambda\Delta t(1-C) & 0 & 1 \end{bmatrix} \begin{bmatrix} P_O(t) \\ P_{FS}(t) \\ P_{FU}(t) \end{bmatrix}$$

System safety

$S(t) = P_O(t) + P_{FS}(t)$

Safety Modeling

$$p_{FS}(t) = C - Ce^{-\lambda t}$$

$$p_{FU}(t) = (1-C) - (1-C)e^{-\lambda t}$$

$$p_O(t) = e^{-\lambda t}$$

$$\longrightarrow \quad S(t) = p_O(t) + p_{FS}(t) = C + (1-C)e^{-\lambda t}$$

time 0 : $S(0) = 1.0$

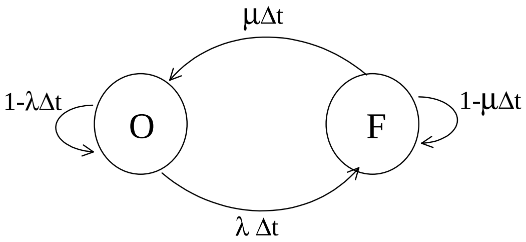
time $\rightarrow\infty$: $S(\infty) = C$

3.6 Availability Modeling

- Previous: failed system stages failed
  Markov models = **acyclic**
- Now: consider system with repair
  return to partial/fully operational status
- Repair Rate (number of repairs within time interval)
  $\mu \cdot \Delta t$ ← probability of repair occurring in time interval Δt
  (system has failed or is inoperable before)

Availability Modeling

Markov chain



Availability Modeling

Corresponding equations

$$\begin{bmatrix} P_O(t+\Delta t) \\ P_F(t+\Delta t \end{bmatrix} = \begin{bmatrix} 1-\lambda\Delta t & \mu\Delta t \\ \lambda\Delta t & 1-\mu\Delta t \end{bmatrix} \begin{bmatrix} P_O(t) \\ P_F(t) \end{bmatrix}$$

- Availability calculation is similar to reliability
- However: repair → cyclic Markov chain

Availability Modeling

Availability for simplex example:

$A(t) = P_O(t)$

Differential equations:

$$\frac{dP_F(t)}{dt} = \lambda P_O(t) - \mu P_F(t)$$

$$\frac{dP_O(t)}{dt} = -\lambda P_O(t) + \mu P_F(t)$$

Availability Modeling

Assume starting state O (operational):

$$P_O(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t}$$

$$P_F(t) = \frac{\lambda}{\lambda+\mu} - \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t}$$

with time t → ∞ $P_O(\infty) - \frac{\mu}{\lambda+\mu} = \frac{1}{1+\frac{\lambda}{\mu}}$ *Steady State Availability*

Availability Modeling

- Remember: A(t) is *"availability"*
  probability that a system will be able to perform its task at time t
- Intuitively: $A(t) = \frac{\text{total operational time}}{\text{total elapsed time}}$
- Availability = percentage of time during which system is operational

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Availability Modeling

- t_{op} total operating time up to t current
- t_{repair} total down/repair time up to t current

$$A(t_{current}) = \frac{t_{op}}{t_{op} + t_{repair}}$$

- If system experiences N failures during its lifetime
  - total operational time = N · MTTF
  - total down/repair time = N · MTTR

Availability Modeling

Steady state availability

$$A_{ss} = \frac{N \cdot MTTF}{N \cdot MTTF + N \cdot MTTR}$$

also: $MTTF = \frac{1}{\lambda}$ $MTTR = \frac{1}{\mu}$

$$A_{ss} = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda} + \frac{1}{\mu}} = \frac{1}{1+\frac{\lambda}{\mu}}$$

Steady State Availability

- e.g. MTTR → 0 very fast repair
  $\Rightarrow A_{ss} \rightarrow 1$
- e.g. $\lambda \rightarrow 0$ very low failure rate
  $\Rightarrow A_{ss} \rightarrow 1$

3.7 Maintenance Modeling

- Maintainability M(t) = probability that a failed system will be restored to working order within specified time
- M(t) = probability that system will be repaired in time $\leq t$
- Repair rate μ
- MTTR = $\frac{1}{\mu}$

Maintenance Modeling

Expression for maintainability
(similar to exponential failure law)

- Start with N systems with a unique fault in each at time t=0
- Later, at time t determine $N_r(t)$, $N_{nr}(t)$ [repaired / not repaired]

$$M(t) = \frac{N_r(t)}{N} = \frac{N_r(t)}{N_r(t) + N_{nr}(t)}$$

Maintenance Modeling

Differentiation

$$\frac{dM(t)}{dt} = \frac{1}{N}\frac{dN_r(t)}{dt}$$

$$\frac{dN_r(t)}{dt} = N\frac{dM(t)}{dt}$$

Rate at which components are repaired

divide LHS by $N_{nr}(t)$

$$\frac{1}{N_{nr}(t)} \cdot \frac{dN_r(t)}{dt}$$

Repair rate function
(we assume constant repair rate μ)

Maintenance Modeling

$$\mu = \left(\frac{1}{N_{nr}(t)}\cdot\frac{dN_r(t)}{dt}\right) = \frac{N}{N_{nr}(t)}\cdot\frac{dM(t)}{dt}$$

$$\frac{dM(t)}{dt} = \mu\,\frac{N_{nr}(t)}{N} \qquad \left[use: \frac{N_{nr}(t)}{N(t)} = 1 - M(t)\right]$$

$$\frac{dM(t)}{dt} = \mu(1 - M(t))$$

$$M(t) = 1 - e^{-\mu t}$$

Closed form
(solving differential equation)

Maintenance Modeling

- If repair rate $\mu \rightarrow 0 \quad \Rightarrow \quad M(t) = 0$
  OK, because then systems cannot be repaired
- If repair rate $\mu \rightarrow \infty \Rightarrow \quad M(t) = 1$
  OK, because then we have “instant” repair

Maintenance Modeling

In Practice

Usually 3 level of repair:

1. Organizational level *(short time)*
   e.g. faults in circuit boards which can be replaced **on site**
2. Intermediate level *(medium time)*
   repairs can be performed in the rear vicinity
   e.g. in EE lab (better than returning product to manufacturer)
3. Factory level (depot level) *(longtime)*
   equipment has to be returned to manufacturer for repair

3.8 Review and Examples

MTTF

- Mean time to failure
- If no repair: expected life time of a component or system
- Expected time value is integral over reliability function

$$E[X] = \int_{o}^{\infty} R(t)dt$$

- During useful life period: $R(t) = e^{-\lambda t}$
  $\mathrm{MTTF} = \frac{1}{\lambda}$

Review and Examples

- Serial case $R(t) = \Pi R_i(t)$

$$MTTF_{Series} = \frac{1}{\sum \lambda_i}$$

- Parallel case $R(t) = 1 - \Pi (1 - R_i(t))$

(for identical comp.)

$$MTTF = \int_o^\infty (1-(1-e^{-\lambda t})^n)dt$$

Review and Examples

Parallel case – different approach:

- Parallel system fails when **all** components fail
- X system life
- X_i component life
- $X = \max \{X_i \ldots, X_n\}$

$$E[X] \approx \frac{1}{\lambda} \sum_{i=1}^{n} \frac{1}{i} \approx \frac{1}{\lambda} \ln(n)$$

Review and Examples

Cold Standby

$$X = \sum_{i=1}^{n} X_i$$

expected life span is sum of component life spans

$$E[X] = \frac{n}{\lambda} = MTTF$$

(much larger than in parallel case)

Review and Examples

TMR

$P_3(t) = e^{-3\lambda t}$

$P_2(t) = 3e^{-2\lambda t} - 3e^{-3\lambda t}$

$P_F(t) = 1 - 3e^{-2\lambda t} + 2e^{-3\lambda t}$

$\Rightarrow R(t) = P_3 + P_2 = 1 - P_F(t)$

$= 3e^{-2\lambda t} - 2e^{-3\lambda t}$

$$E[X] = \int R(t)dt = \frac{5}{6\lambda} = MTTF_{TMR}$$

anything unusual?

Review and Examples

$$E[X] = \int R(t)dt = \frac{5}{6\lambda} = MTTF_{TMR}$$

anything unusual?

$$\boxed{MTTF_{TMR} < MTTF_{component}}$$

very surprising!

- Where does this come from ??
- Paradox comes from looking only at mean value!
- Reliability is much higher in the beginning for TMR and becomes less later