

Autonomous Hexacopter Software Design

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Abstract

Multicopter unmanned aerial vehicles are a popular up and coming platform for autonomous robotics research. This report outlines the design and implementation of a multithreaded software system for a hexacopter controlled by a raspberry pi.

An array of sensors including a GPS, an IMU and a camera are used on the hexacopter, and the software system is responsible for collecting data from and monitoring these sensors through their lower level libraries. A modular approach is taken to software design so that the system can be used in a variety of applications.

The applications tested in this project were: GPS waypoint navigation and single coloured object tracking. GPS navigation was achieved with a maximum path deviation of 5.66m. A discussion on improvements to the navigation algorithm and the incorporation of course correction methods is provided. Object tracking algorithms were implemented using center of mass and camshift techniques, which were optimized using HSV thresholding, lookup tables and reduced colour spaces. The hexacopter was able to track a red object moving at a slow waking pace for a full trial, 1.13 minutes in length.

The software system presented is flexible, currently in use with other simultaneous projects and sets out a framework for future development.

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1. Introduction and background

1.1 Introduction

Multirotor UAV (unmanned aerial vehicles), namely quadcopters and hexacopters, have become increasingly popular in recent years. Like the similar single rotor helicopters, they possess the ability to hover on the spot but have the added advantage that they are far more maneuverable; they can move in directions left and right just as well as forwards and backwards. The multirotor technology is becoming more popular and viable for industrial applications as the battery technology used to power the copters becomes lighter, lasts longer and becomes more cost effective. The large hobbyist following has also made the technology widely accessible to researchers.

UAVs are currently being used in several industrial applications. In the mining industry they are used for aerial mapping, surveying and mining exploration [1]. In the oil and gas industry they are used in both onshore and offshore settings for flare stack, pipeline and structural inspections [2]. Similarly, they are also used in the power industry for transmission line inspection [3]. Each of these applications takes advantage of the large areas of operation and the ability to remove workers from hazardous environments.

However, multirotor technology has only recently become viable for industrial uses and there is still much room for research into the many possible applications for this technology.

1.2 Motivation

The agricultural industry is one of the application areas barely touched upon by UAV technology. Given the large, open and obstructionless areas involved, UAVs are well suited to applications in this industry.

It is a known fact that a plants health can be told by its thermal image. Plant leaves absorb most wavelengths of visible light, but reflect green light and also the non-visible near infrared light. The reflection of green light is attributed to chlorophyll and other pigments, while infrared reflection is from the internal structure of the

plants leaves [4]. Unhealthy plants reflect less infrared light and this difference can be detected remotely using a thermal imaging camera.

Causes for unhealthy plants include: disease, pest infestation, frost and water stress [5]. This leads to practical applications, for example: pesticides can be selectively applied only to affected regions and irrigation blockages can be detected remotely and fixed early.

UAVs are being used in conjunction with thermal cameras in an agricultural setting [6], but the process is yet to be automatized. The motivation for this project is to combine the two technologies of autonomous flight and intelligent image processing to produce a machine that will identify areas of interest in real time and carry out autonomously inspection.

1.3 Background

This project is building onto the efforts of Venables [7] and O'Connor [8], who in 2013 constructed the hexacopter from a hobbyist kit and demonstrated its capabilities in autonomous flight with onboard navigation and image processing algorithms.

This year's project was adopted by myself and 3 other students. The project is split into 4 components:

- Myself – Core software system involving interactions with sensors and actuators.
- Omid Targhagh – GPS based navigation including trajectory planning and intelligent search patterns.
- Merrick Cloete – Onboard image processing and object detection.
- Alex Mazur – Web based user interface and integration of all parts.



Figure 1.3.1 The UWA hexacopter.

1.4 Goals / design problem

The problem addressed by this part of the project is the lack of software framework. The code written last year had a single purpose and was difficult to expand and maintain. The goal of this project is to develop a software system that is flexible and robust, and to demonstrate effectiveness through applications.

2. Literature review

Multicopter UAVs are a keen platform for many autonomous aerial robotics research projects. Of the literature surveyed, research divides into either indoor applications, mainly employing a vision based navigation system, or outdoor applications using GPS based navigation. Common across the literature are the 5 parts of an autonomous aerial robotic system:

- 1) Sensors and hardware
- 2) System modeling
- 3) Pose estimation and sensor fusion
- 4) Control loops
- 5) Goal setting

2.1 Sensors and hardware

Nearly all of the projects surveyed used an IMU (inertial measurement unit) [9-17], a device that contains accelerometers, gyroscopes, magnetometers and an embedded controller to combine readings to provide information on the UAV's roll, pitch and yaw, as well as accelerations in each direction. The acceleration data can be integrated to provide positional data, but as discussed in many papers [10, 15, 17], the accumulation of errors makes readings unreliable without supplementation with other sensors.

GPS (global positioning system) is most preferred for outdoor applications [13, 15, 17] and provides a positional fix. GPSs suffer from GPS drift, whereby the accuracy of positional data is restricted to a window and this window shifts due to the number of GPS satellite signals received and the relative positions of the satellites [15]. This problem is solved either by combining data with that from an IMU [10, 15, 17] or by using a differential GPS system [10, 13]. In the differential GPS system, two GPSs are used, one fixed and the other roving. The GPS drift that effects both can be canceled out, provided they are kept close enough together and share access to the same satellite signals.

Vision based navigation systems usually employ either a single camera [9] or stereo cameras [10-12]. In both cases, a technique called optical flow is used for visual odometry. It involves identifying features in a frame and tracking how much their positions change throughout successive frames. With knowledge of how far away the features are and the frame rate, the copters speed is estimated [10, 11]. Stereo vision lends itself to calculating the distance of the features from the cameras using perception [10], while other distance sensors such as ultrasound are needed for single camera systems [9].

A few systems use more advanced mapping systems such as laser scanners [12] and infrared motion sensors [16]. These are used in SLAM (simultaneous localization and mapping) systems, whereby the copter navigates relative to its surroundings, mapped out by scanners.

2.2 Modeling

Most copter project have involves kinematic modeling the copter system. The extent of modeling varies across the literature. The most basic models are based on the equations of Newtonian motion [9, 15-17]. For a hovering copter, the force of upward thrust is equal to its weight. The copter moves by adjusting roll and pitch, such that a component of thrust is in the horizontal plane. Other projects use much more advanced models, incorporating fluid dynamics [17], blade flap effects [13] and gyroscopic effects [18].

A novel approach to modeling was taken by [14], in which the tuning process was automated. The copter was set up in an environment with VICOM infrared tracking cameras, so that the copter's pose was digitally known at all times. Starting from the first principles model, the copter was instructed to do a flip. The controlling computer then readjusted the model parameters to match the observed path and the process reiterated until an accurate model was determined.

2.3 Pose estimation and sensor fusion

A problem identified across the literature was that sensors are never completely accurate; the system must estimate its position and orientation (pose) using the best combination of data available to it. This is solved using a sensor fusion algorithm, often a variation on the Kalman filter.

The Kalman filter is a two step algorithm. First the next kinematic state of the system is predicted using a model (as described above) and the previous system state. Secondly, the state is corrected based on data from the sensors, taking the expected error of each sensor into account and weighting appropriately using a least squares regression method. The process repeats so that the estimated state is constantly refreshed and corrected [19].

The most popular sensor fusion algorithm used across the literature surveyed is actually the EKF (Extended Kalman Filter) [9-13, 15-17]. The EKF reflects the fact that most systems are non-linear and cannot be described by the linear model required by the regular Kalman filter. The EKF uses a non-linear model, but takes the first order Taylor expansion to make it fit into the Kalman filter framework [19].

2.4 Control loops

Traditional PID controllers have been used for both copter stability [13, 14] and for velocity regulation [9, 13, 14]. Modern control techniques including state variable feedback [16, 20] and linear quadratic regulator systems [18] have also be used and found satisfactory.

2.5 Goal setting

Higher level goal setting algorithms have been implemented as finite state machines [9, 12]. The high-level control loop cycles through states representing goals. Alternately, [16] uses a path manager and represents paths as edges on a graph, which is incorporated into its SLAM program.

3. System Overview

3.1 Hardware system

The hexacopter hardware is mostly unchanged since work done in 2013 [7, 8], except for the upgrade from a raspberry pi model B to a model B+. A brief overview is as follows: a commercial hexacopter, the DJI flamewheel [21], was purchased, built from a kit and serves as an experimental platform. This kit includes a flight board that contains accelerometers and stabilization algorithms. Under normal operation, RF (radio frequency) signals are sent from the handset's transmitter to the RF receiver, converted to PWM signals and sent to the flight board.

Apart from stabilization, all other autonomous control algorithms are run on the raspberry pi. The raspberry pi is programmed to output PWM signals mimicking those produced by the RF receiver. A relay switch was installed to switch between the two PWM signals, and is controlled by one of the channels of the RF receiver. The throttle channel (altitude) is not passed through the switch and the operator is in control of the aircraft's altitude at all times. This was done as a safety precaution.

The raspberry pi is fitted with an array of sensors, including: a GPS, an IMU and a CMOS camera. An overview of the hardware system is outlined in figure 3.1.1. A list of hardware is presented in table 3.1.2.

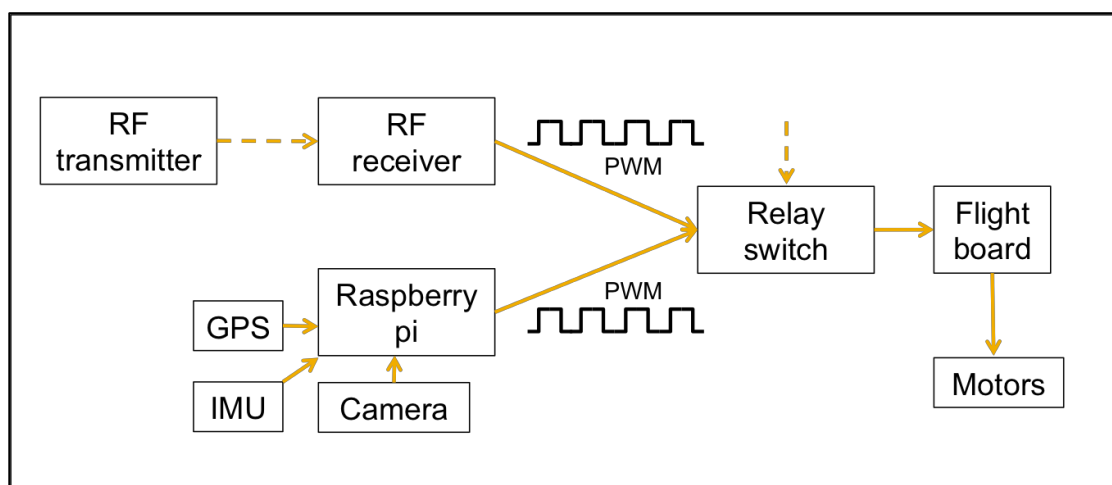


Figure 3.1.1 Simplified diagram of hardware system. Not all channels are passed through the relay switch; the throttle channel (altitude) is kept under manual control.

Hardware	Brand / model	Communication protocol	Library
Flight board	DJI / NAZA-M [22]	PWM	ServoBlaster [23]
GPS	Qstarz / BT-Q818X [24]	NMEA-0183	wiringPi [25]
IMU	Xsens / MTi [26]	Proprietary serial over usb	Xsens CMT3 [26]
Camera	Raspberry pi / Camera module [27]	Serial over 15pin ribbon cable	raspicamCV [28]

Table 3.1.2 List of hardware on hexaopter with communication protocols and software libraries.

A new PIKSI brand differential GPS [29] was trialed for the system. The manufacturers claim centimeter positional accuracy, however the product is still in its infancy and the firmware contains too many bugs to be used effectively.

A recent addition to the hexaopter is a low voltage piezo buzzer. This presents the option of audio feedback for when the copter achieves a goal set out by the top-level control algorithm.

Communication hardware includes: an analogue video transmitter, a wifi dongle and a 4G dongle. These are discussed in section 3.3.

3.2 Software system

In contrast to previous work on this project, a multithreaded approach was taken to designing the software system. Each sensor has its own background thread that constantly reads and updates incoming data, whilst monitoring for errors and logging. This approach was much inspired by Drage’s work on autonomous cars [30]. There is an emphasis on separating the mid-level system (incorporating data collection and monitoring of sensors) from the top-level or application level controller, which is responsible for navigation algorithms and user interactions.

The advantage of this design is that top-level threads can block while waiting for user input without halting the data gathering threads. Also, this approach is necessary for integration with the web-based user interface (designed by Mazur), since GPS and

orientation data needs to be accessible to both the control systems and for display on the user interface.

Communication with the flight board was achieved by outputting PWM on the GPIO pins using the ServoBlaster library written by Hirst [23]. The flight board does not provide feedback to the raspberry pi. Instead, a copy of the values sent to each channel is kept and it is assumed that they have been passed to the flight board correctly. The user does not manipulate the pulse widths of the signals directly; rather a value is specified between -100 to 100 representing the % power to the channel. For example 100 on the elevator channel is full power forwards. The values are then scaled to the appropriate pulse widths.

The GPS is monitored by reading and parsing the character stream read in using the wiringPi library written by Henderson [25]. The GPS stream is of NMEA [31] format and an example is provided below. The second, third and fifth values (separated by commas) are time, latitude and longitude respectively. Other information is also offered in the NMEA string including the number of satellite signals received and horizontal dilution; see [31] for more details.

```
$GPGGA,044130.400,3158.7227,S,11548.9609,E,1,5,11.99,23.8,M,-29.4,M,,*69
```

The GPS has its own thread that constantly runs in the background, parsing NMEA messages for appropriate information. The thread also monitors the GPS, and enters an error state when the GPS is disconnected, doesn't receive any data after a timeout period or loses satellite fix. This thread runs at approximately 5Hz, to match the data rate of the GPS.

The Xsens IMU communicates over RS242 serial over USB through proprietary CMT3 serial libraries [26]. There were issues with the default data rate being too high; running the thread at the matched rate caused the thread to hog the CPU resources, while slowing it down caused data to build up in a buffer (implemented by the CMT3 library), causing data to be old. Changing the mode of the IMU to decrease the data transmission rate and running the thread at a matched rate solved this problem.

The IMU was calibrated using the MagfieldMapper program provided by Xsens [26]. The process involved rotating the IMU, whilst in place on the copter, around in as many different orientations as possible. MagfieldMapper calibrated the IMU internally against the acceleration due to gravity and the magnetic environment.

Additionally, all image processing is conducted in a self-contained background thread (the camera is treated as a sensor for coloured objects). Image processing is done using the OpenCV software libraries [32] and Valkov's RaspiCamCV library [28]. The design criterion for the camera thread is to have the best possible frame rate for a decent resolution. The focus of the algorithms discussed in section 5 is on efficiency. Effectiveness and more complex image detection algorithms analyzed in the part of the project undertaken by Cloete.

The buzzer is operated using a signal from one of the GPIO pins. When activated, a thread is detached and simulates PWM by turning the pin on and off at frequencies of up to 100Hz. GPIO manipulation is done using the wiringPi library [25]. This was chosen over ServoBlaster so that the frequency can be altered without affecting the pins used for flight board communication. Although the piezo buzzer technically plays sounds at a single pitch, changing the frequency and pulse width can produce a noticeable variety of chirps at different volume levels.

3.3 Communication system

Three methods of in-flight communication with the raspberry pi were used: direct connection using Bluetooth keyboard and analogue video transmitter, remote connection using SSH over a wifi network and communication to a web server over either wifi or 4G.

Direct communication using the Bluetooth keyboard was the primary communication method used in 2013 [7]. It involves logging into the raspberry pi on the copter the same as if it were a desktop computer. In place of a monitor, a video transmitter is used to send the analogue video to a pair of headset goggles. The goggles can be plugged into a monitor via RWY connection. Commands are given to the copter

through the terminal. Camera vision is streamed by running the X window system and displaying images on the desktop. This method is effective for up to 20m, the approximate range of the Bluetooth keyboard.

A similar approach is to remotely log in to the raspberry pi using SSH (secure shell hosting). This does, however, require a wifi network present, which can either be provided by a wireless router out on the field, or through an ad hoc network created by the wifi dongle. Commands are sent through the remote terminal. Camera feed can be streamed across SSH, but this is slow and laggy (~2fps with a lag of ~3s). This method can be used for distances up to 30m away, depending on the wifi dongle.

The final communication method to the pi is via the web interface designed by Mazur for his part of the project. This user friendly interface simplifies communication to running only a handful of application programs including waypoint following and area searching and the user defines by dropping markers onto a map. Camera streaming is available and is incorporated into the camera thread. OpenCV's inbuilt video writer is used to write the camera feed to a mjpg (using method in [33]) that is then incorporated into the web server using mjpg-streamer [34, 35]. The web server can also be set up and used over 4G when the wifi dongle is replaced with a 4G dongle.

The multi-threaded software system is made robust by using mutexes to protect data as it is passed from the sensor threads to the controller threads. The modular design satisfies the flexibility criterion. The effectiveness of the software system is tested through applications in both GPS-based and camera-based navigation.

4. GPS based navigation

4.1 Distance and bearing calculations

It should be noted that a convention that bearing is measured clockwise from north is used. This was chosen to align with the flight board's convention, whereby a rotation right is defined positive.

The flat earth approximation is one method of finding the distance and bearing between two latitude-longitude coordinates [36]. The assumption made that, in the neighborhood of the coordinates, the earth is flat, i.e. longitudes are parallel and bearing does not change along a straight path. The E-W and N-S axes are mapped to the Cartesian x and y axes respectively. Equations 4.1.1-4 are the flat earth approximation equations.

$$x = R \times (\text{lon}2 - \text{lon}1) \times \cos(\text{lat}1) \times \frac{\pi}{180} \quad \text{Eqn 4.1.1}$$

$$y = R \times (\text{lat}2 - \text{lat}1) \times \frac{\pi}{180} \quad \text{Eqn 4.1.2}$$

$$d = \sqrt{x^2 + y^2} \quad \text{Eqn 4.1.3}$$

$$\theta = \text{atan2}(y, x) \quad \text{Eqn 4.1.4}$$

Where R is the radius of the earth, d is the distance between the two coordinates and θ is the bearing between from coordinate 1 to coordinate 2.

The haversine distance and great circle bearing methods are more accurate over larger distances [37]. The shortest distance between two coordinates is actually along a great circle (path with diameter of entire earth). Equations 4.1.5 and 4.1.6 are the haversine distance and great circle bearing formula respectively.

$$d = 2R \sin^{-1} \left(\sqrt{\sin^2 \left(\frac{\text{lat}2 - \text{lat}1}{2} \right) + \cos(\text{lat}1) \cos(\text{lat}2) \sin^2 \left(\frac{\text{lon}2 - \text{lon}1}{2} \right)} \right) \quad \text{Eqn 4.1.5}$$

$$\theta = \text{atan2}(\sin(\text{lon2} - \text{lon1}), \cos(\text{lat1}) \sin(\text{lat2}) - \sin(\text{lat1}) \cos(\text{lat2}) \cos(\text{lon2} - \text{lon1}))$$

Eqn 4.1.6

For the scale of distances covered by a hexacopter in autonomous flight, the difference between the two methods is negligible. For example, taking two GPS coordinates of neighboring Perth suburbs : (32.071 S, 115.825 E) and (32.888 S, 116.010 E), the haversine formula yields 26.78km, while using flat earth approximation yields 26.79km, a difference of 0.04%.

4.2 Direct flight vector method

A flight vector is defined as the horizontal direction and magnitude of the thrust the hexacopter applies to its motors.

Direct flight vectoring methods was used in the 2013 project [7] and was reimplemented on the new software system. Using haversine and great circle bearing equations, the distance and bearing from the copter's current GPS coordinate and the next waypoint is calculated. Speed is controlled using a PI controller on distance, and bearing is always in the direction towards the next waypoint. The speed saturates at a maximum safe autonomous speed. There is a dead-band around the waypoint such that the copter is to stop before the waypoint and use its momentum to drift towards the waypoint. The waypoint is triggered when the copter enters the dead-band.

This method has an issue that the P controller decreases the speed to a point that the copter stops before it triggers the waypoint, particularly when the copter misses and takes a second pass. For that reason the I component is introduced. The accumulator is cleared every time a waypoint is reached. A D controller is not necessary because the system relies on overshoot to reach the waypoint.

Figure 4.2.1 shows the speed and direction the copter is to take using this method and the proportional component only.

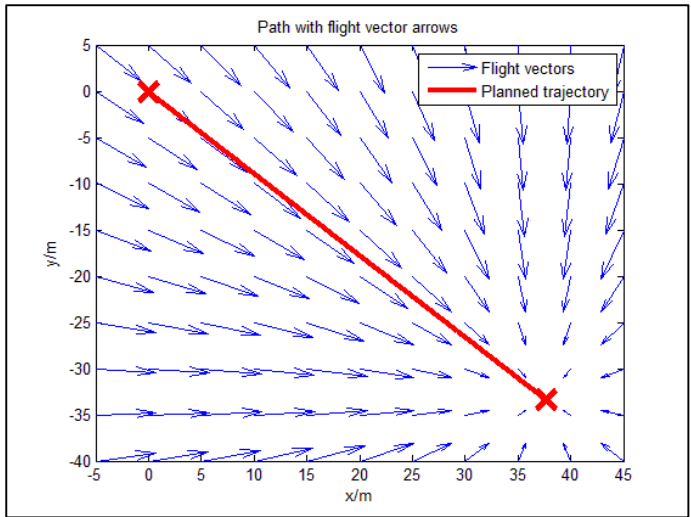


Figure 4.2.1 Simulation of flight vectors on path from (31.9797S, 115.89177) to (31.9800S, 115.8181E), with origin at (31.9797S, 115.89177). $K_p = 1$.

4.3 Waypoint following algorithm

When navigation to waypoints, the copter is set to strafe without changing its yaw (heading). The copter can be facing any direction; the yaw is subtracted from the desired bearing produce the relative bearing from the copter's frame of reference.

Figure 4.3.1 shows the control flow loop used for waypoint navigation.

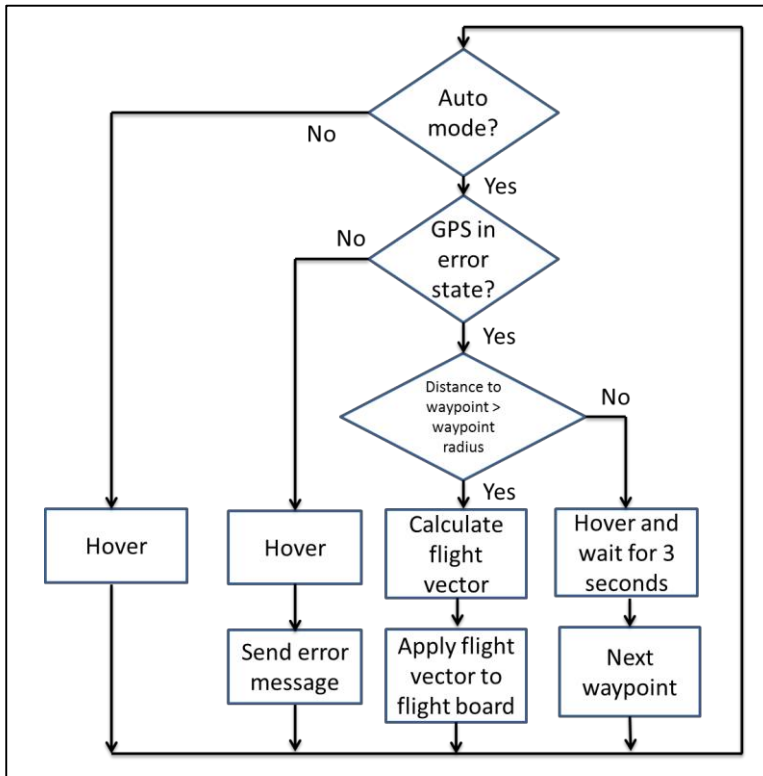


Figure 4.3.1 Control flow diagram for waypoints following algorithm

5. Camera based navigation

The goal of image processing is to find efficient algorithms for image processing. Image processing is done using OpenCV image processing libraries. The flaw with this method is that all processing is done on the raspberry pi's single core processor, which acts as a bottleneck when reading in and processing images.

5.1 HSV thresholding

HSV (hue, saturation, value) thresholding is used for coloured object detection (typically a red test object). Hue is the position of the colour on the colour wheel (see figure 5.1.1). Starting from black, value is the amount of white added to the mix and saturation is the amount of the colour added to the mix. For the red test object (hue from 350° to 10°), the upper right corner of the saturation-value square is of interest (figure 5.1.2). Learning colours is then trivial; only the hue thresholds need be altered to match the average hue over an area.

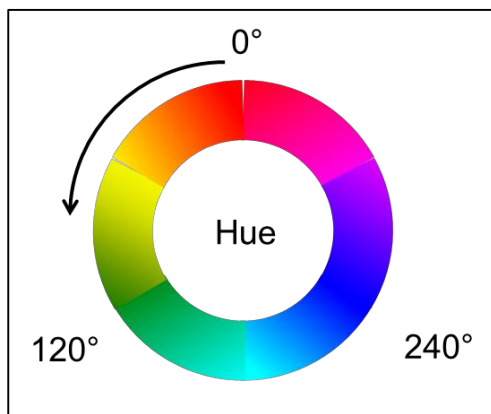


Figure 5.1.1 Hue colour wheel

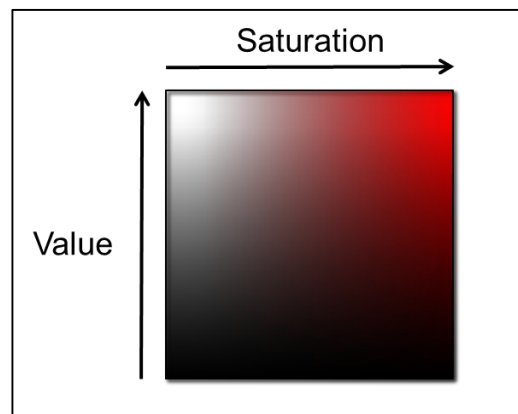


Figure 5.1.2 Saturation-value square

Conversion from RGB to HSV is time costly. To speed the process up, HSV thresholding has been implemented using a lookup table. During the initializing and setup stage, for every R, G and B combination the HSV equivalent is calculated and a value of '1' is stored in the array if the colour is red or '0' otherwise. Retrieving an array entry is very quick, but comes with the costs that the thresholds cannot be changed without recalculating the lookup table and that the array is quite large (256x256x256 bytes).

To offset the large memory cost the colour space has been reduced. Each channel is mapped from [0,255] down to buckets of [0,7]. This conversion is also done using a lookup table.

5.2 Single coloured object detection

A center of mass type algorithm is used for coloured object detection. The average position of the red pixels is calculated as the sum of rows or columns of the red pixels divided by the total number of red pixels. If the number of red pixels is larger than a certain threshold, a red object is considered as being detected.

The flaw with this algorithm is when more than one red object is present in the frame; the average is split between the objects. An improvement on is the cam-shift algorithm [38]. Rather than scanning over the whole frame, only a window is scanned. Assuming that a red object is present in the frame and moves slow enough that it does not escape the window entirely, the center of mass is recalculated and the window for the next frame is constructed around the new center of mass. That way the window follows the object.

If the window is of a fixed size, the algorithm is called a mean-shift [38]. For an adaptive window, the algorithm is a cam-shift (continuously adaptive mean-shift) and, using the assumption that the object is square, the length and width of the window are made proportional to area of the object, or the number of pixels detected. In the implementation of this algorithm on the copter, the window is set to the entire frame when no objects are detected.

5.3 Coloured object tracking

The following program loop was used for tracking a red object (figure 5.3.1). The copter hovers, rotates on the spot and lowers/raises the gimbal to keep the object in frame.

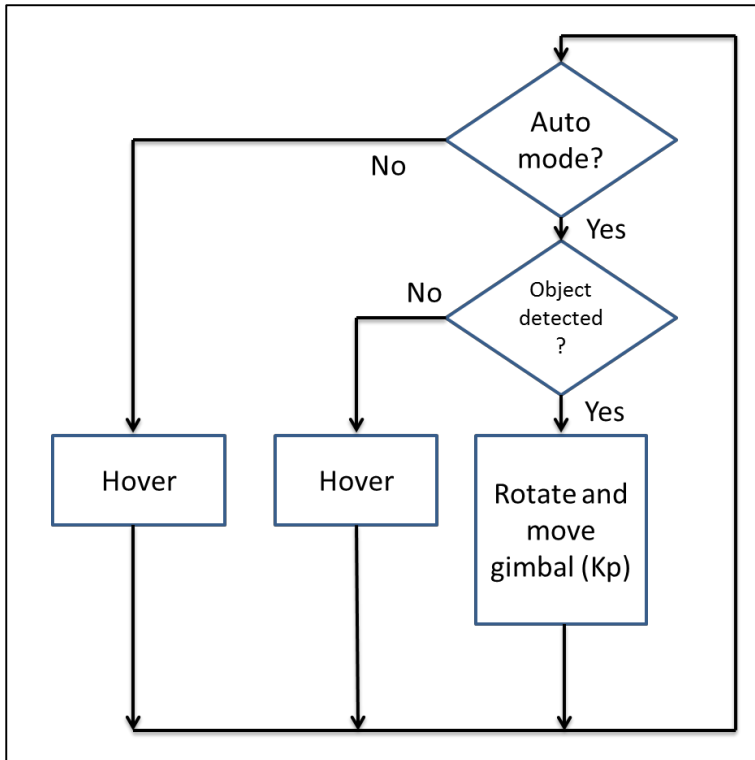


Figure 5.3.1 Control flow diagram for object tracking

6. Results and discussion

6.1 GPS-based navigation

Trials were done at UWA's James oval over the cricket pitch. As a safety precaution, no one is to walk under the hexacopter whilst in flight. The cricket pitch was used for convenience because the grounds people barricad it off to prevent people from walking on it.

Figures 6.1.1a and 6.1.2a show the copters desired and recorded trajectories whilst following sets of two and four waypoints respectively. Figures 6.1.1b and 6.1.2b show the perpendicular deviation from the path against time for the two runs. The average and maximum deviations are shown in table 6.1.3.

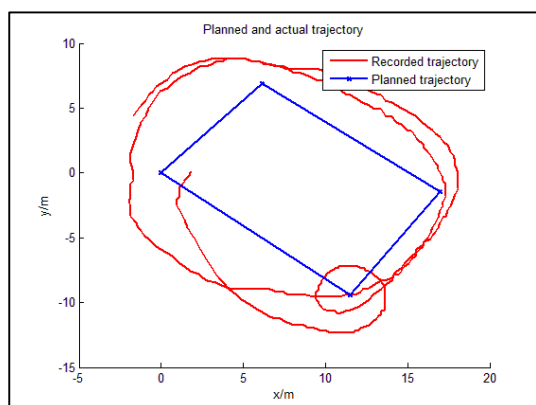


Figure 6.1.1a *Planned and recoded trajectories for 4 waypoints*

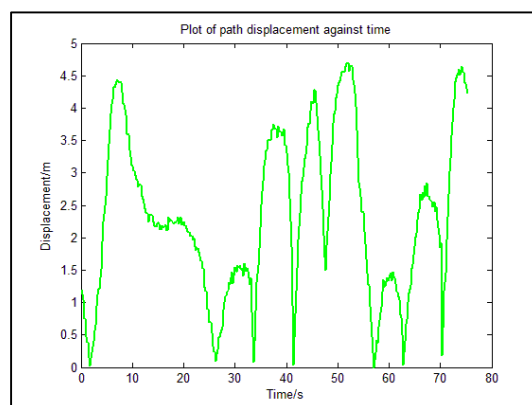


Figure 6.1.1b *Plot of path deviation against time for run in Fig 6.1.1a*

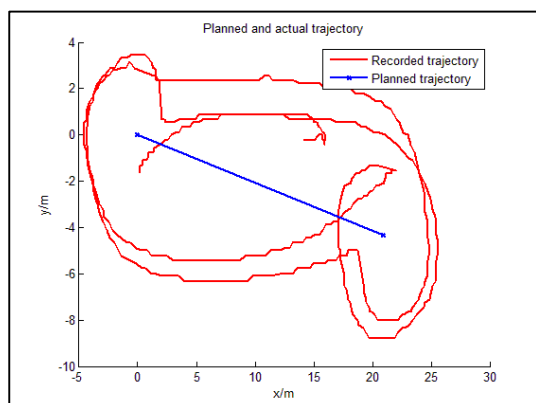


Figure 6.1.2a *Planned and recoded trajectories for 2 waypoints*

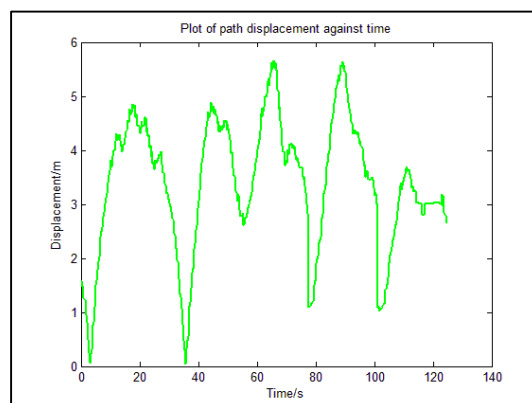


Figure 6.1.2b *Plot of path deviation against time for run in Fig 6.1.2a*

	Average deviation	Maximum deviation
Run 1	2.33 m	4.70 m
Run 2	3.39 m	5.66 m

Table 6.1.3 Summary of results from testing: Average and maximum deviations from path.

The maximum deviation from path was 5.66m, which is comparable to the results from 2013 (4m) [7].

These results can be improved by using one of the course correction algorithms proposed in section 7.

6.2 Camera based navigation

Figures 6.1.1 and 6.1.2 show the center of mass and camshift algorithms tracking a red test paddle. As can be seen in figures 6.1.3 and 6.1.4, the center of mass algorithm is easily broken by introducing a second paddle, whereas the camshift algorithm continues tracking the first paddle.



Figure 6.1.1 Center of mass algorithm.

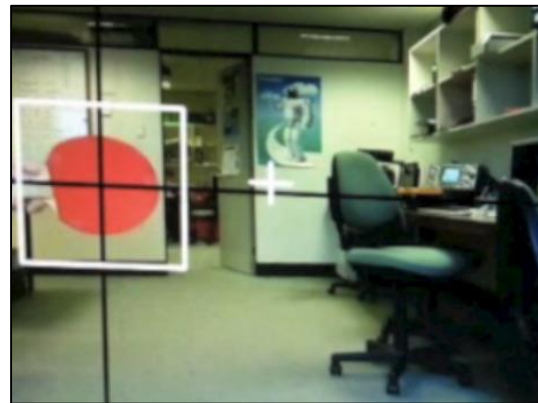


Figure 6.1.2 Camshift algorithm.

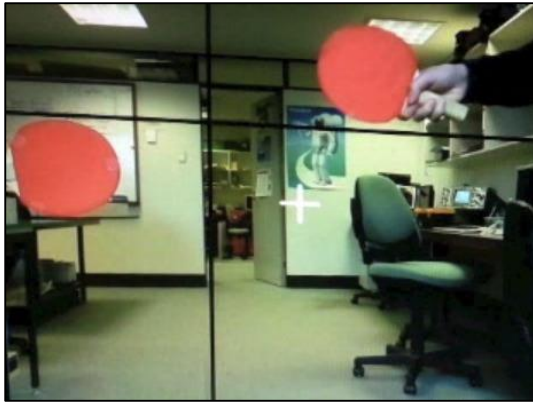


Figure 6.1.3 Center of mass algorithm with second red object introduced.

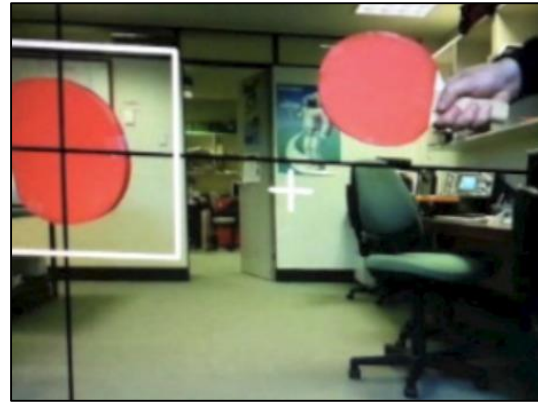


Figure 6.1.4 Camshift algorithm with second red object introduced.

The main goal of this part of the project is to increase the efficiency of image processing. Experiments were conducted as follows: the program was run in isolation exhaustively and timed how long it took to capture process and display 10000 frames, and the average frame duration is taken as a measure of efficiency (the shorter, the more efficient). Frame rate is found by taking the inverse of average frame duration.

Figure 6.2.5 is a comparison of capturing and displaying different resolution images without any image processing algorithms. Using the smallest resolution (320x240) produced the best results.

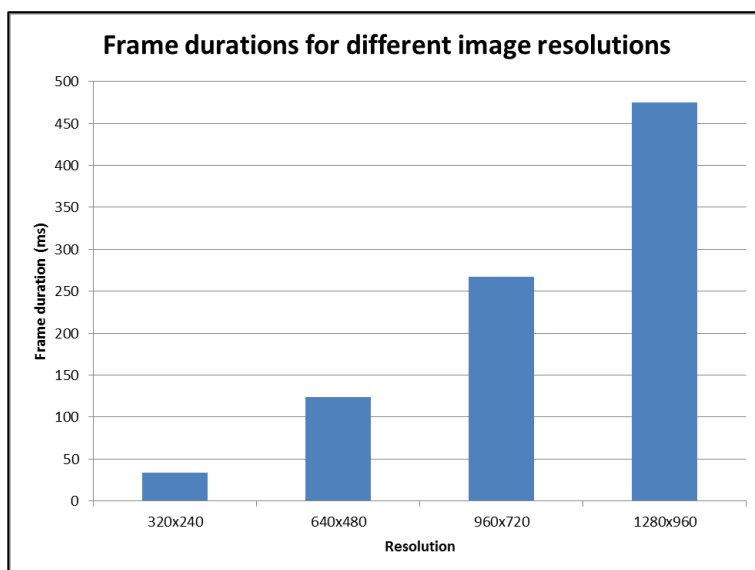


Figure 6.2.5 Frame durations for capturing and displaying images at different resolutions. No image processing conducted in this set of tests.

Figure 6.2.6 is a comparison of the different image processing algorithms proposed in section 5.2. The resolutions listed are virtual resolutions; resolution was decreased by skipping pixels when processing without creating a new image. All images are still captured and displayed at the minimal 320x240 resolution. The algorithm performs satisfactorily when images are processed at a resolution of 160x120.

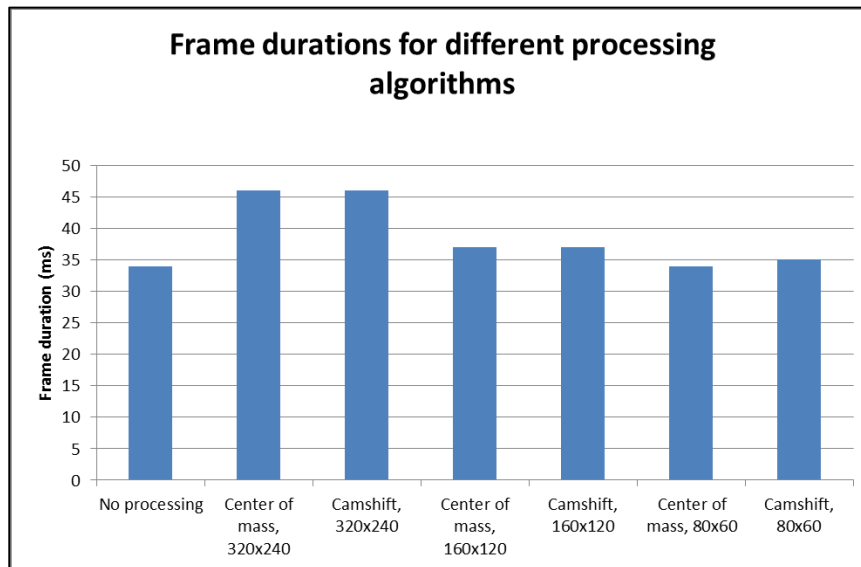


Figure 6.2.6 Frame durations for capturing, processing and displaying images. All images are captured and displayed at 320x240. The resolutions listed are those of the image processing algorithm.

Figure 6.2.7 is a comparison of the image processing whilst being streamed as an mjpg. All images are captured at a resolution of 320x240 and processed at a resolution of 160x120. Streaming at 160x120 produced a satisfactory video stream for viewing on a tablet or smart phone.

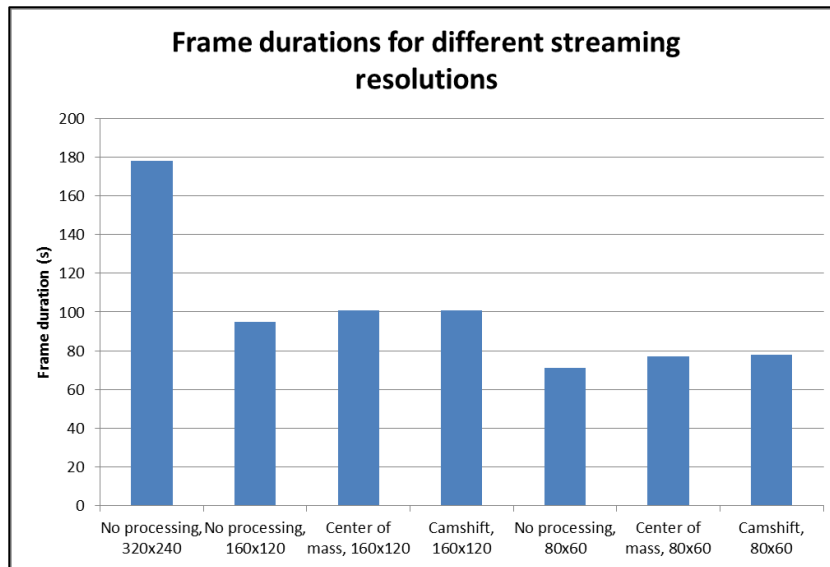


Figure 6.2.7 Frame durations for capturing, processing and streaming images. All images are captured at 320x240 and processed at 160x120. The resolutions listed are those of the image stream.

For a reasonable resolution of 320x160, images can be processed using the camshift algorithm with resolution of 160x120 at a frame rate of up to 26fps. This is a considerable improvement from 2013 where the maximum frame rate achieved with a similar algorithm was 8.5fps [7].

This process can also be streamed with a resolution of 160x120 at 9.8fps.

Using the flow program devised in section 5.3, the copter was able to track a red bin lid moving at slow walking pace (~1kph) for 1.13min, the entire length of the trial. The video is hosted at:

<<https://dl.dropboxusercontent.com/u/68798381/Hex%20Demo.mp4>>

6.3 Discussion of practical issues and problems encountered when using the raspberry pi on a hexacopter platform

Several practical issues have arisen when using the current hardware setup. These range from power to vibration issues.

The main issue with using a commercial hexacopter kit is the lack of feedback from the platform. There is helper software provided by DJI for calibration and troubleshooting, however, the amount of support is limited. For example, at the start of the year when the copter had not been flown over the summer break, the flight board required recalibrating and refused to take flight until the process was complete. The helper software was however unable to communicate that as the problem.

In other cases, problems have been unintuitive to diagnose. For example, at one point the hexacopter was acting extremely unstable even when under manual (handset) control. It could not hold its position and would start to spiral around a spot until it became uncontrollable, despite all channels being kept at neutral. The problem was that the DJI compass (attached to the flight board) had been twisted during a crash the previous week. This was unintuitive, since any compass problems should have been observable through yaw corrections.

Power issues are the main practical problems with using the raspberry pi on the hexacopter. The raspberry pi cannot supply enough power when using both the GPS and IMU and while using the wifi dongle to route a network. As a result, the IMU was not used for many of the tests. The alternate method used was to fly forwards for a set period of time and find use the bearing of the path travelled as the copters yaw. Of course, any turbulence causes error in the yaw and the copter misses waypoints by a few degrees, resulting in a circling pattern. Two proposed solutions are to either install a powered hub on the copter, drawing off the batteries, or to replace the Xsens IMU with a low power alternative, since the Xsens IMU is not specifically designed for a hexacopter application and only the compass data is used by the current navigation algorithms.

A final notable problem when using the hexacopter comes from the large amount of vibrations created by the motors. There was one case during testing when using a microSD to SD card converter and the vibrations caused the microSD card to dislodge during flight. This caused loss of access to the file systems, although this was not catastrophic since the programs run from RAM.

7. Alternative flight vector methods

The following alternate flight vectoring methods were designed and simulated. A crash late in the project prevented them from being tested.

7.1 Intermediate goal flight vectoring

To add course correction capabilities, a number of intermediate goals or path points are plotted, equally spaced apart and in a line between the previous waypoint and the last. A path point is counted as reached either if the copter comes within a predefined distance of the path point, or if the next path point is closer than the current one.

The speed of the flight vector is calculated using a PI controller on distance to the last path point or next waypoint, same as in 4.2. This was done rather than making speed proportional to the distance to the closest path point so that the copter does not slow down for each path point.

The bearing of the flight vector is calculated as a linear combination of bearings to all unreached path points. Many path points are used to calculate bearing to prevent oscillations due to the large changes in bearing that occur when the copter is close to the path point.

Bearings are summed using the weighted average angle formula (adapted from [39]):

$$\bar{\alpha} = \text{atan2}(\sum_{i=0}^n w_i \cdot \sin(\alpha_i), \sum_{i=0}^n w_i \cdot \cos(\alpha_i)) \quad \text{Eqn 7.1.1}$$

Where n is the number of angles, α is the angle and w is the weight for each angle:

$$w_i = \frac{1}{i} \quad \text{Eqn 7.1.2}$$

The path points are weighted such that each progressive path point is weighted less than the previous one.

Figure 7.1.1 is a quiver plot showing the flight vectors using the above method and a P controller only. The main feature is course correction.

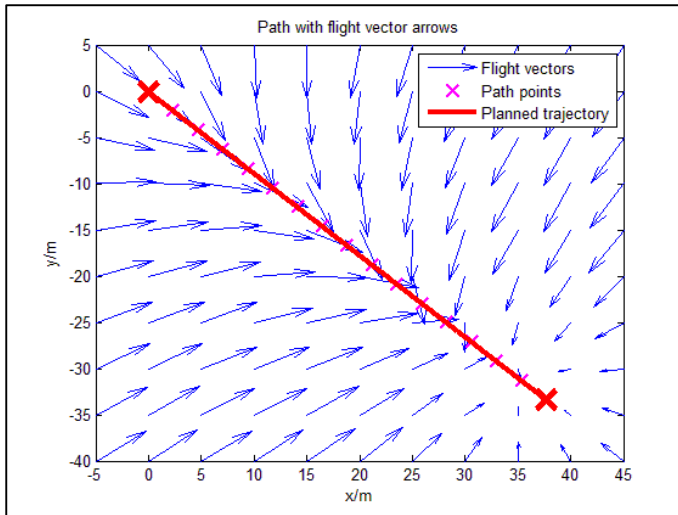


Figure 7.1.1 Simulation of flight vectors on path using intermediate goal method.
 $K_p = 1$.

7.2 Improved direct flight vectoring

In the previous flight vectoring methods, the only one controller is used and the direction it acts on changes depending on the relative positions waypoints to the copter. An improved version is proposed by using two orthogonal controllers, one acting on the N-S axis and another on the E-W axis.

Using the flat earth approximation, lat/lon coordinates are transformed into Cartesian coordinates (using the copter's location as the origin). A waypoint is represented by vector X_0 :

$$X_0 = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \quad \text{Eqn 7.2.1}$$

And the hexacopter is represented by vector H :

$$H = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{Eqn 7.2.2}$$

Therefore the error matrix contains each orthogonal component in separate rows:

$$e_x = H - X_0 = \begin{bmatrix} -x_0 \\ -y_0 \end{bmatrix} \quad \text{Eqn 7.2.3}$$

This method is more intuitive, since the memory of the PID controller has a notion of direction, although this has reduced effect since the copter is flying to a position

rather than maintain one. For a P controller only, this method yields the same result as the first direct flight vectoring method. Again, the controller's memory is cleared when it reaches the waypoint.

Figure 7.2.1 shows the flight vectors using the above method and a P controller only.

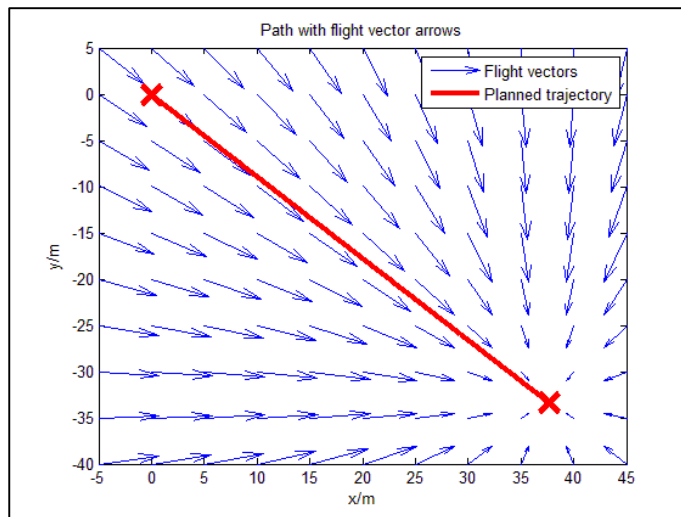


Figure 7.2.1 Simulation of flight vectors on path using improved direct flight vectors. $Kp_x = Kp_y = 1$.

7.3 Line following flight vectoring

Similar to the method proposed in 7.1, a path between two waypoints is plotted as a straight line. Two orthogonal controllers are used; one on the distance perpendicular to the path and one on the inline distance to the next waypoint. The perpendicular controller keeps the copter on path, while the inline controller controls speed.

This algorithm again works using the Cartesian coordinate system (using the previous waypoint as the origin). The line between two points is defined parametrically:

$$l : P(t) = a_1 t + a_0 \quad \text{Eqn 7.3.1}$$

$$a_0 = X_0 \quad \text{Eqn 7.3.2}$$

$$a_1 = X_1 - X_0 \quad \text{Eqn 7.3.3}$$

Where X_1 is the next waypoint and X_0 is the previous one. t is only the linking parameter for the line such that $P(t=0)$ is the start of the line segment and $P(t=1)$ is the end. t does not represent time.

After plotting the path as a line segment, the next part of this method is projecting the copter's position onto the path. This is taken as the closest point on the line to the copter, point $P(t_1)$. This is the point where vectors $HP(t_1)$ and X_1X_0 are perpendicular, and t_1 is when the dot product of the two vectors is zero.

$$(H - P(t_1)) \cdot (X_1 - X_0) = 0 \quad \text{Eqn 7.3.4}$$

Working backwards, t_1 is found. The side point H is on the line is found using the sign of determinant:

$$|H - X_0 : X_1 - X_0| = \begin{vmatrix} H_x - X_{0x} & X_{1x} - X_{0x} \\ H_y - X_{0y} & X_{1y} - X_{0y} \end{vmatrix} \quad \text{Eqn 7.3.5}$$

Where the determinant is positive when the H is on the right hand side of line X_0X_1 and negative when it is on the left.

The perpendicular and inline errors are calculated as:

$$e_{perpendicular} = -sgn(|H - X_0 : X_1 - X_0|) \cdot \|H - P(t_1)\| \quad \text{Eqn 7.3.6}$$

$$e_{inline} = sgn(t - 1) \cdot \|X_1 - P(t_1)\| \quad \text{Eqn 7.3.7}$$

Where the Euclidian norm is used to calculate distances.

The system must then be rotated to the global frame of reference by adding the following angle to the bearing:

$$\theta = atan2(X_{1y} - X_{0y}, X_{1x} - X_{0x}) \quad \text{Eqn 7.3.8}$$

Figure 7.3.1 shows the flight vector plot using this method. This system has course correction like that in section 7.1.

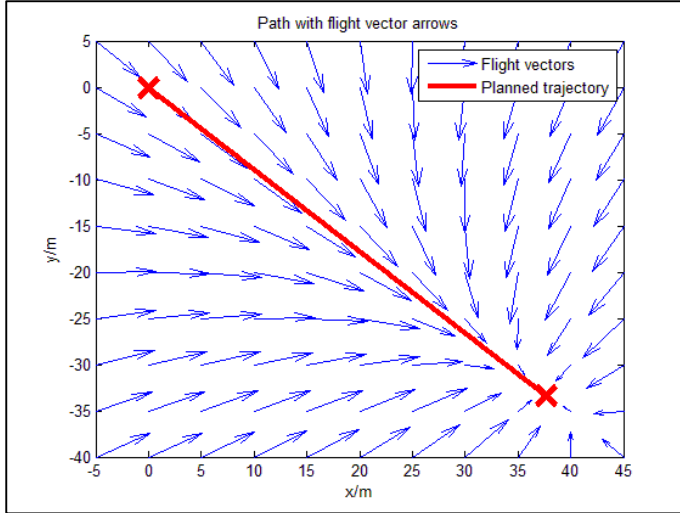


Figure 7.3.1 Simulation of flight vectors on path from using line following algorithm. $Kp_{inline} = 1$, $Kp_{perpendicular} = 1$.

7.4 Curve following

The last navigation technique proposed is to follow a simple curve. Curve following give the added feature that the copter does not need to stop at waypoints; rather it slows down to make turns and passes smoothly through all waypoints.

The approach taken is extended from the straight-line method above. Paths are made by interpolating quadratic curves between groups of three waypoints. Higher order polynomial splines could be used, but are kept to the 2nd order for simplicity.

The parabola is defined parametrically, such that $P(t=0) = X_0$, $P(t=1) = X_1$ and $P(t=t_1) = X_2$ with $t_1 > 1$. The following equations are used [40]:

$$c : P(t) = a_2 t^2 + a_1 t + a_0 \quad \text{Eqn 7.4.1}$$

$$a_0 = X_0 \quad \text{Eqn 7.4.2}$$

$$a_1 = X_1 - X_0 - t_1 \quad \text{Eqn 7.4.3}$$

$$a_2 = \frac{(X_1 - X_0) - t_1(X_1 - X_0)}{t_1(t_1 - 1)} \quad \text{Eqn 7.4.4}$$

Given that X_0 , X_1 and X_2 are defined by the waypoints list, the last defining constant is t_1 , which is visualized as how much further X_2 is along the path from X_0 than X_1 . In the proposed navigation method, only the portion of the curve from X_0 to X_1 ($t = [0, 1]$) is used. Once X_1 is reached, a new curve is calculated using X_1 , X_2 and the next waypoint.

Traditional spline techniques have the requirement that for an n th order spline, the $n-1$ derivatives of polynomial segments are equal at the start and end points of the segment. The copter, however, is capable of strafing in any direction so paths are not required to be completely smooth, only continuous. Instead, it is proposed that the quadratic curve is designed such that the point X_1 is at the ‘apex’ or the symmetry point of the parabola. This is so that the large bend does not occur on the used part of the curve ($t: 0 \rightarrow 1$) and to simplify the problem of finding the closest point of the path to the copter’s position.

Using the symmetry property of parabolas, it is approximated that $|1-t|$ is roughly proportional to the distance of $P(t)$ from the ‘apex’ (the assumption being that X_1 is at the ‘apex’), and this is used to define t_1 :

$$\frac{\|X_1 - X_0\|}{1-0} = \frac{\|X_2 - X_1\|}{t_1 - 1} \Rightarrow t_1 = 1 + \frac{\|X_2 - X_1\|}{\|X_1 - X_0\|} \quad \text{Eqn 7.4.5}$$

The above assumption holds as long as the distances between waypoints are of the same order of magnitude. Figure 7.4.1 shows the paths plotted for a set of four waypoints. Only the solid parts of the curves are part of the path. The dashed part is the continuation of the parabolas to X_2 .

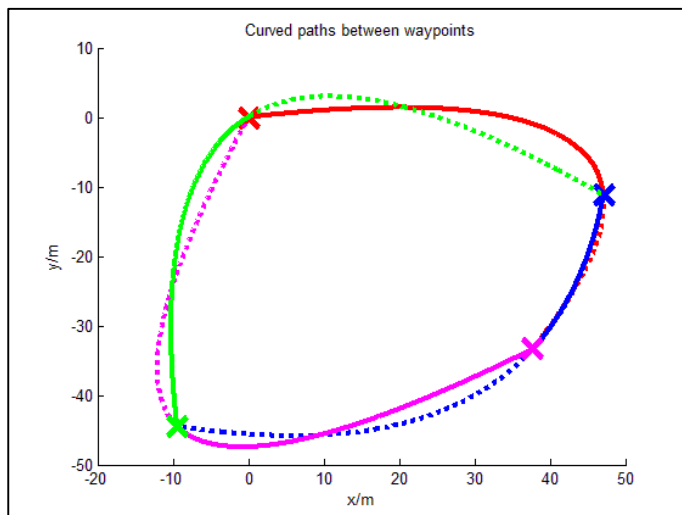


Figure 7.4.1 Quadratic curve paths for a set of 4 waypoints. A clockwise path is taken.

Now that the set of rules for planning a path given a set of 3 waypoints has been defined, the next part of this method is an algorithm for projecting the copters position onto the path. Based on the assumption that the symmetry point of the parabola is at $t=1$, there will only be one point $P(t_2)$ with $t_2 < 1$ where the distance between the copter's location and the curve is at a minimum. Therefore the closet point on the curve to H can be found using a binary search.

The search is restricted to 10 iterations so that the time taken on this step is roughly constant, and gives t_2 to a resolution of 0.001, which translates to a 10m path being broken up into 1cm sections.

The next part is to find which side of the path the copter lays. A line segment tangential to the path is extended from point $P(t_2)$ to point Q:

$$Q = P(t_2) + \frac{\partial P(t_2)}{\partial t} \cdot 1 = P(t_2) + 2a_2t_2 + a_1 \quad \text{Eqn 7.4.6}$$

And equation 7.3.5 is again used to calculate which side of the path the point lays on.

The perpendicular distance is calculated using the Euclidian distance between H and $P(t)$, same as above:

$$e_{perpendicular} = -sgn(|H - P(t_2) : Q - P(t_2)|) \cdot \|H - P(t_2)\| \quad \text{Eqn 7.4.7}$$

Course correction is achieved using a PID controller on the above error.

It is also desired that the copter only slows for waypoints without actually stopping. The amount it slows down should depend on the angle between waypoints $X_0X_1X_2$, i.e. the copter should stop for 180° turns but pass through linear points (0° turn) at full speed. The speed controller should only begin to take effect when the copter is a predefined distance from the waypoint. The proposed speed controller equation is set out below:

$$speed = \begin{cases} MAX & dist > approach \\ MAX + (MAX - MIN) \left(\sin\left(\frac{\theta}{2}\right) \right) \left(1 - \frac{dist}{approach} \right) & o/w \end{cases} \quad \text{Eqn 7.4.8}$$

Where MAX is the maximum speed, MIN is the minimum speed, θ is the angle between waypoints, $dist$ is the straight line distance to the waypoint and $approach$ is the predefined distance of approach when the copter is to start slowing for a turn.

Figure 7.4.2 is a plot of the flight vectors on a curved path using the above method. Points towards the right of the figure aren't populated with vectors since it is assumed that the copter is following the new path it plotted after it passed the second waypoint.

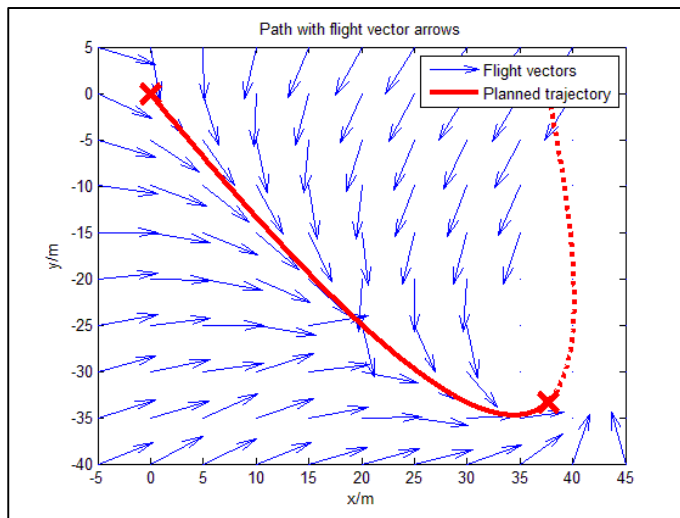


Figure 7.4.2 Simulation of flight vectors on path from using curve following algorithm.

8. Conclusions and future work

8.1 Conclusions

A multithreaded software system was designed for and implemented on an autonomous hexacopter. The modular design made this system flexible and adaptable. The use of mutexes to protect resources across threads makes the design robust.

The software system was proved to successfully control a hexacopter in autonomous flight. GPS waypoint navigation was achieved with a path deviation comparable to last year. Camera image processing streamlined, effectively doubling previous frame rates. Finally the object tracking was sustained for 1.13 minutes, the entire length of the test.

This system is currently in use with a variety of simultaneous projects, including field searching and multiple object tracking, all tied in together with a web based user interface.

8.2 Future work

Modeling of the system is a priority. This will allow for efficient tuning of PID parameter, the application of state estimation algorithms, namely the EKF, and even integration with robotics simulators.

Finally, the hexacopter must be fitted with a thermal imaging camera to achieve the projects original goal: an autonomous application in the agricultural setting.

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